TABLE 2. Values of α/α_0 at Various Compressions
Neglecting Possible Phase Changes
Calculations made by using equation 19 and 29.

V/V_0	Al_2O_3 ($\beta_T = 4.00$)	Single-Crystal MgO ($\beta_T = 4.54$)	Mg_2SiO_4 ($\beta_T = 4.78$)	α Quartz ($\beta_T = 6.3$)	Garnet ($\beta_T = 5.44$)
0.95	0.815	0.792	0.782	0.724	0.757
0.9	0.656	0.620	0.604	0.515	0.564
0.8	0.410	0.363	0.344	0.245	0.297
0.7	0.240	0.198	0.182	0.106	0.143
0.5	0.063	0.043	0.036	0.013	0.023

Published values of the three quantities in (28) for single-crystal MgO, polycrystalline Al_2O_3 Mg_2SiO_4, and single-crystal garnet measured in the author's laboratory, are shown in Table 1. The ratio defined by (28) is, in fact, close to unity. The fact that it is not exactly unity is not significant because of the uncertainty in the value of α, which affects δ_{S0} and γ_0 in an opposing way. What is significant is that the last column of values in Table 1 is close enough to unity to lend credence to the following approximation:

$$\delta_{S0} + \gamma_0 \cong \delta_T \cong \beta_T \tag{29}$$

Temperature effects on α at high pressure become very easy to handle if the following holds:

$$\left(\frac{\partial}{\partial T}\right)_P \left(\frac{\partial}{\partial P}\right)_T B_T = \left(\frac{\partial \beta_T}{\partial T}\right)_P = 0 \tag{30}$$

There is considerable experimental evidence that equation 30 holds for nonmetallic solids at high temperature (see *Anderson* [1966*c*]). In this case equation 17 is replaced by

$$\begin{aligned} \ln\left[\frac{\alpha(P, T)}{\alpha(0, T)}\right] &= \frac{-\delta_T}{\beta_T} \ln\{1 + \beta_T[P/B_T(0, T)]\} \\ &= \delta_T \ln\left[\frac{V(P, T)}{V(0, T)}\right] \end{aligned} \tag{31}$$

The change of α due to a change in phase at a given P-T can be computed by means of equation 31, but the values of $\alpha(0, T)$, $B_T(0, T)$, and β_T are appropriate to the new phase.

DISCUSSION

To compare our equation for α with that presented by *Birch* [1952], equation 17 is expanded by series and the first terms are retained. In this approximation, equation 17 is

$$\alpha(P, T_0)/\alpha(0, T_0) = 1 - \delta_T P/B_T(0, T_0) \tag{32}$$

The above is the same as Birch's equation for α at low pressure. To obtain Birch's equation for high pressure, replace $B_T(0, T_0)$ by $B_T(P, T_0)$. Taking δ_T and $B_T(0, T_0)$ as arbitrary constants, equation 32 is equivalent to the formula presented by *Jacobs* [1952, 1953].

The circumstantial evidence in favor of the Murnaghan equation of state is now so abundant that the law of expansivity based on the Murnaghan equation of state seems reasonably well founded.

We now turn to the use of equation 26 for computations of planet interiors, for which the parameters are unknown because of a lack of knowledge of the chemical composition and phase of the planet interiors. The question can be phrased as 'What values of δ_T are appropriate to phases in the planet interior?' As an approximation we use equation 29 since data on β_T are more often available than data on δ_T. The answer, therefore, is that the value of δ_T ought to be restricted between the extreme values found for β_T for oxide compounds: the lower limit should be $\beta_T = 4.0$, as found for Al_2O_3, and the upper limit $\beta_T = 6.3$, as found for α quartz [*Anderson*, 1966*a*].

It turns out that the ratio of α/α_0 for compression of single phases up to about 0.5 (which corresponds to the pressure at the earth's center) can be constrained between the limits 0.063 and 0.013. Calculations given for the five solids listed in Table 1 are presented in Table 2.

In view of the uncertainty of the nature of the composition of the earth's mantle, it should be sufficient to assume that the expansivity is inversely proportional to the bulk modulus at all depths, where the bulk modulus is computed from the seismic velocities, $B_S = \rho(v_p^2 - 4/3\, v_s^2)$, and where the values of $\alpha(0, T_0)$ are given appropriate to assumptions of composition in regions B and D of the earth's mantle.

Note added in proof: Y. A. Chang (*J. Phys. Chem. Solids, 28*(4), 697–701, 1967) just published a relationship between δ and β_T. His equation 19 is the same as equation 15 of this paper, if q is taken to be zero, and if γ_0 is replaced by unity in equation 27.

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